day 41

3.7/97

$$\lim_{x\to 2} \frac{(x^2-3)^5-1}{x-2}$$

one of the limit def^n of a deriv

$$f'(2)=\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}$$

$$f(x)=(x^2-3)^5$$

$$f'(x)=5(x^2-3)^4\cdot\frac{d}{dx}(x^2-3)$$

$$=5(x^2-3)^4(2x)$$

$$f'(2)=5(2^2-3)^4(2\cdot 2)=20$$

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3.10/6

Explain how to find $(f^{-1})'(y_0)$ given that $y_0 = f(x_0)$

$(f^{-1})'(y^0) = \dfrac{1}{f'(f^{-1}(y_0))}$

$(x_0, y_0) \xrightarrow{f^{-1}(x)} (y_0, x_0)$

need $f^{-1}(y_0)$!

But - if $y_0 = f(x_0)$ THEN $f^{-1}(y_0) = x^0$

$$\frac{1}{f'(x_0)}$$

$f(f^{-1}(x)) = x$

$f'(f^{-1}(x)) \cdot (f^{-1})'(x) = 1$

$(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$

3.9 - Logarithmic Differentiation

day 41

$\ln(ab) = \ln a + \ln b$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$\ln a^b = b \ln a$

EXAMPLE

$$f(x) = \frac{(x^3-1)^4\sqrt{3x-1}}{x^2+4}$$

preliminary . . .

$$\ln(f(x)) = \ln\left(\frac{(x^3-1)^4\sqrt{3x-1}}{x^2+4}\right)$$

$$\ln(f(x)) = \ln(x^3-1)^4 + \ln\sqrt{3x-1} - \ln(x^2+4)$$

$$\ln(f(x)) = 4\ln(x^3-1) + \tfrac{1}{2}\ln(3x-1) - \ln(x^2+4)$$

$$\ln(f(x)) = 4\ln(x^3-1)$$

end preliminary

take derivative - remember the Ala - chain rule - mo

$$f(x)\left[\frac{1}{f(x)}\cdot f'(x)\right] = \left[4\left(\frac{1}{x^3-1}\right)(3x^2) + \frac{1}{2}\left(\frac{1}{3x-1}\right)(3) - \frac{1}{x^2+4}(2x)\right]f(x)$$

$$f'(x) = f(x)\left[\frac{12x^2}{x^3-1} + \frac{3}{2(3x-1)} - \frac{2x}{x^2+4}\right]$$

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If $f(x)$ is a function, AND

$f^{-1}(x)$ is also a function [[no domain crippling]]

then $f(x)$ is SAID to be
ONE-TO-ONE.

every x-value in the domain has only 1 value;
and every y-value in the range is "pointed to"
by a single point in the domain.

$f(x)$ is "invertable"

Remember

$f(x)$ | $f^{-1}(x)$
DOMAN | DOMAIN (formerly known as range)
RANGE | RANGE (formerly known a domain)

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38/52

$x+y^3-y=1$; at $x=1$ [[so $1+y^3-y=1$

day 41

$y^3-y=0$

$y(y-1)(y+1)=0$

$y=-1,0,+1$]]

$\frac{d}{dx}(x)+\frac{d}{dx}(y^3)-\frac{d}{dx}(y)=\frac{d}{dx}(1)$

$1+\left(3y^2\frac{dy}{dx}\right)-\frac{dy}{dx}=0$

$\frac{dy}{dx}(3y^2-1)=-1$

$\frac{dy}{dx}=\frac{-1}{3y^2-1}$

at $(1,0)$

$\frac{dy}{dx}=\frac{-1}{3(0)^2-1}=+1$

at $(1,-1)$

$\frac{-1}{3(-1)^2-1}=\frac{-1}{2}$

at $(1,1)$, see

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3.8/81)

$$x^2 \cdot x^2 + x^2 y^2 + x^2 \cdot x + y^2 x^2 + y^2 y^2 + y^2 x = 8xy^2$$

$$x^4 + 2x^2y^2 + x^3 + y^4 + xy^2 = 8xy^2$$

so $x^4 + 2x^2y^2 + x^3 + y^4 = 7xy^2$ then take deriv

Moral: sometimes simplifying first helps

[[if you are good at algebra]]

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3.9/45)

day 41

$f(x) = x^{\cos x}$; $a = \frac{\pi}{2}$

$\ln f(x) = \ln(x^{\cos x})$

$\ln(f(x)) = \cos x \cdot \ln(x)$

$\frac{1}{f(x)} \cdot f'(x) = (-\sin x)\ln x + \cos x\left(\frac{1}{x}\right)$

$f'(x) = x^{\cos x}\left[-\sin x \ln x + \frac{\cos x}{x}\right]$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

$\frac{d}{dx}\ln[f(x)] = \frac{1}{f(x)} \cdot f'(x)$

Y?

I know the rule if x is in base

I know the rule if x is in exponent

I know NUDDIN if x is in both places

day 41

3.9/6

$$b^x = \underbrace{e^{\ln b^x}}_{\text{I can do this with anything}} = \underbrace{e^{x \ln b}}_{\text{looking JUST at the exponent....}}$$

$\ln b^x = x \ln b$

IMPORTANT note about logarithms

$a^b = x \Leftrightarrow \log_a x = b.$

the logarithm is ONLY an exponent